Uncertainty Analysis of Calculated Extinction From Apparent Contrast Measurements From Images of Natural Targets

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1.0 Introduction

The contrast of a given feature at a specific wavelength of light is determined by the radiance of that feature and the radiance of the background or adjacent feature against which it is viewed. Target/Sky contrast *(C)* is defined as the ratio of the difference between the feature (target) and the background sky radiance to the background sky radiance:

$$C = \frac{\left({}_t N - {}_s N\right)}{{}_s N} \tag{1}$$

where N stands for radiance, t and s represent target and background sky respectively. Two important definitions are:

Inherent Contrast:	C_o	The contrast of the target at zero distance
Apparent Contrast:	C_r	The contrast of the target as viewed at a distance r

For an atmospheric sight path with average an extinction coefficient = b_{ext}, the apparent contrast (C_r) to an observer who is viewing an object of inherent contrast (C_o) is rigorously defined:

$$C_r = C_o\left(\frac{{}_s N_o}{{}_s N_r}\right)\exp\left(-b_{ext} r\right) \tag{2}$$

where ${}_sN_o$ and ${}_sN_r$ represent the radiance of the identical sky reference position as measured at the target location (distance = 0*)* and at the observation point (distance = *r)* respectively (Malm, 1979).

Equation 2 can be rearranged to solve for b_{ext}:

$$b_{ext} = \frac{1}{r}\ln\left(\left(\frac{{}_s N_o}{{}_s N_r}\right)\times\left(\frac{C_o}{C_r}\right)\right) \tag{3}$$

Equation 3 can be simplified by defining a sky radiance ratio as $R_N = {}_sN_o / {}_sN_r$:

$$b_{ext} = \frac{1}{r}\ln\left(\frac{R_N C_o}{C_r}\right) \tag{4}$$

Thus the average extinction coefficient of the sight path can be calculated from measurements of apparent contrast *(C_r,)* and target distance *(r)* provided estimates of the inherent contrast *(C_o)* and sky radiance ratio *(R_N)* are made. The question then is:

What is the uncertainty associated with calculated b_{ext} when employing contrast measurements from images (35mm transparencies or digital) of natural targets and using equation 4?

2.0 Propagation of Uncertainty

The relative precision (uncertainty U_x) of the mean *(X)* of a set of measurements of some variable is defined as (Watson et. al., 1983):

$$U_x = \frac{SIG_x}{X} \tag{5}$$

where: SIG_x = standard deviation of measurements

Applying standard procedures to equation 4, the propagation of uncertainty through it can be expressed as (Watson et. al., 1983):

$$U_{bext}^2 = U_r^2 + \left(\frac{\left(U_{R_N}^2 + U_{C_O}^2 + U_{C_r}^2\right)}{\ln^2\left(\frac{R_N C_o}{C_r}\right)} \right) \tag{6}$$

where:

U_{bext} = Relative uncertainty of calculated extinction
U_{CR} = Relative uncertainty of the apparent contrast measurement
U_r = Relative uncertainty of the sight path distance measurement
U_{RN} = Relative uncertainty of sky radiance ratio estimate
U_{CO} = Relative uncertainty of inherent contrast estimate

Equation 6 assumes that there are no correlations between any of the variables. Taking the uncertainty in r as zero (we should be able to measure the sight path distance with a very high degree of precision) the equation 6 can be simplified to:

$$U_{bext} = \left(\frac{\left(U_{R_N}^2 + U_{C_O}^2 + U_{C_r}^2\right)^{0.5}}{\ln\left(\frac{R_N C_o}{C_r}\right)} \right) \tag{7}$$

Equation 7 shows that the actual values and uncertainties in R_N, C_o and C_r are all equally important and cannot be ignored when estimating the uncertainty in calculated b_{ext}.

2.1 Relative Uncertainty in Sky Radiance Ratio *(R_N)* Estimates

Typically when calculating b_{ext} from target/sky contrast measurements, the sky radiance ratio is not measured but is assumed to be equal to 1.0. The validity of this assumption has been investigated in numerous field experiments and modeling efforts and has been found to essentially never be true in practice (Malm et. al., 1986; Malm and Henry, 1987),. However, since R_N cannot be measured from an image (color slide or digital) the assumption $R_N = 1$ is typically employed. Using information from field measurements and modeling efforts, the relative uncertainty in R_N can be reasonably assumed to be about 30%:

$$U_{R_N} = 0.3 \tag{8}$$

2.2 Relative Uncertainty in Inherent Contrast *(C_o)* Estimates

In estimating b_{ext} from target/sky contrast measurements, the inherent contrast is not measured but is estimated from historical analyses of the cleanest days at a monitoring site. C_o is a function of the illumination of the target, solar/observer geometry, and the background extinction. Thus, C_o is varies with time of day, time of year, cloud cover, target surface reflectance, and ambient extinction. Inherent contrast can vary from -0.2 for a granite cliff in direct solar illumination with a backscattering solar geometry to -1.0 for dark coniferous forests shaded by clouds with a bright forward scattering background sky. In practice, a standard C_o is estimated for a target at a particular time of day from the cleanest days available. This C_o is then used in all subsequent b_{ext} calculations. Analyses of historic teleradiometer and slide contrast data have indicated that the uncertainty in the C_o estimates for reasonably dark targets (tree covered ridges) is typically 10%, 20% for grass covered targets, and 40% for bare rock (Malm et. al., 1981; Malm et. al., 1982; Malm and Molenar, 1984):

U_{CO} = 0.1 for tree covered targets
U_{CO} = 0.2 for grass covered targets
U_{CO} = 0.4 for bare rocks

2.3 Relative Uncertainty in Apparent Contrast *(C_r)* Measurements

The apparent target contrast is calculated from the density measurements made on 35mm color slides or analysis of calibrated digital images.

For color slides this requires a transformation of measured slide densities to the exposures on the slide which are directly related to the onsite radiances by employing published film response curves. The uncertainty of C_r from 35 mm slides has been examined by comparing measured slide C_r from slide densitometry versus simultaneous onsite radiometric measurements of C_r (Johnson et. al., 1985 and Dietrich, et. al., 1989). The analysis indicates that the uncertainty is about ± 0.04 in contrast. This means that the relative uncertainty varies from over 100% for targets just barely visible to about 10% on very clean days:

$$U_{C_r} = \frac{0.04}{C_r} \tag{9}$$

For digital images this requires a proper calibration of the image capture device (Berns, 2001).

3.0 Method For Assigning Uncertainties To b_{ext} From Slide Contrast Measurement

The above analyses lead to the following method for estimating the uncertainty in b_{ext} calculated from contrast measurements from images:

1 - assume $R_N = 1.0$ and $U_{RN} = 0.3$

2 - estimate C_o from clean days for each time of day
and for a maximum error estimate, assign uncertainties as:

U_{CO} = 0.1 dark targets
U_{CO} = 0.2 light targets
U_{CO} = 0.4 bright targets

3 - employ an estimated absolute measurement precision
for C_r of 0.04 thus:

$$U_{C_r} = \frac{0.04}{C_r} \tag{10}$$

then for any measured C_r the equations for calculating b_{ext} and its associated uncertainty are:

$$b_{ext} = \frac{1}{r}\ln\left(\frac{C_o}{C_r}\right) \tag{11}$$

$$U_{bext}(\%) = 100\left(\frac{\left(0.09 + U_{C_O}^2 + \frac{0.0016}{C_r^2}\right)^{0.5}}{\ln\left(\frac{C_o}{C_r}\right)}\right) \tag{12}$$

4.0 Example Calculations

Figure 1 is a plot of the extinction uncertainty equation for three typical sites in the US Forest Service monitoring network that represent a wide variation in natural target characteristics:

Quail Prairie, Oregon	Dark Target	$C_O = -0.90$
Hercules Glade, Missouri	Light Target	$C_O = -0.70$
Sawtooth, Idaho	Bright Target	$C_O = -0.50$

It is apparent that the lowest uncertainty approaches 25% only for dark targets and that is always greater than 4*0%* for bright rock targets.

Figure 2 plots the estimated uncertainty in calculated b_{ext} from slide densitometry of a 30 km distant target and the estimated uncertainty for b_{ext} from transmissometry along a 5.0 km path length (Molenar et. al., 1992) as a function of true ambient extinction. Three target characteristics are plotted, dark, light, and bright using the previous inherent contrasts. In the b_{ext} range of 20 to 100 Mm^{-1}, which is typical of class I areas in the western United States, the transmissometer extinction uncertainty is less than 15%. The uncertainty associated with slide contrast measurements is in the range 30 to 100% of true b_{ext}. A 10% change in extinction is equivalent to one deciview *(dv)***,** a perceptible change in visual air quality. The 5.0 km path transmissometer typically has uncertainties less than ± 1.5 *dv*, while slide contrast measurements have uncertainties in the range ± 3 to 10 *dv!*

5.0 References

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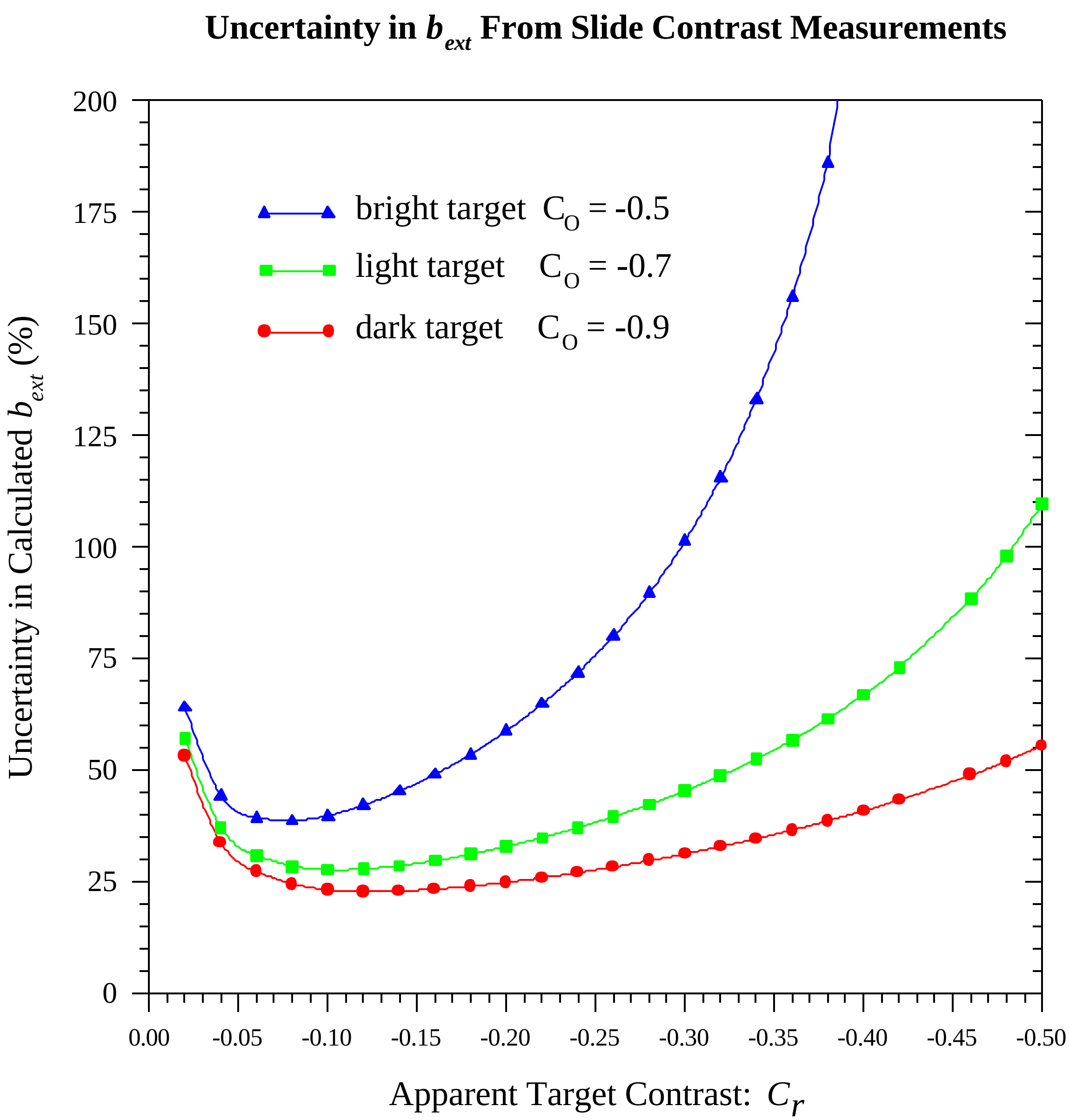
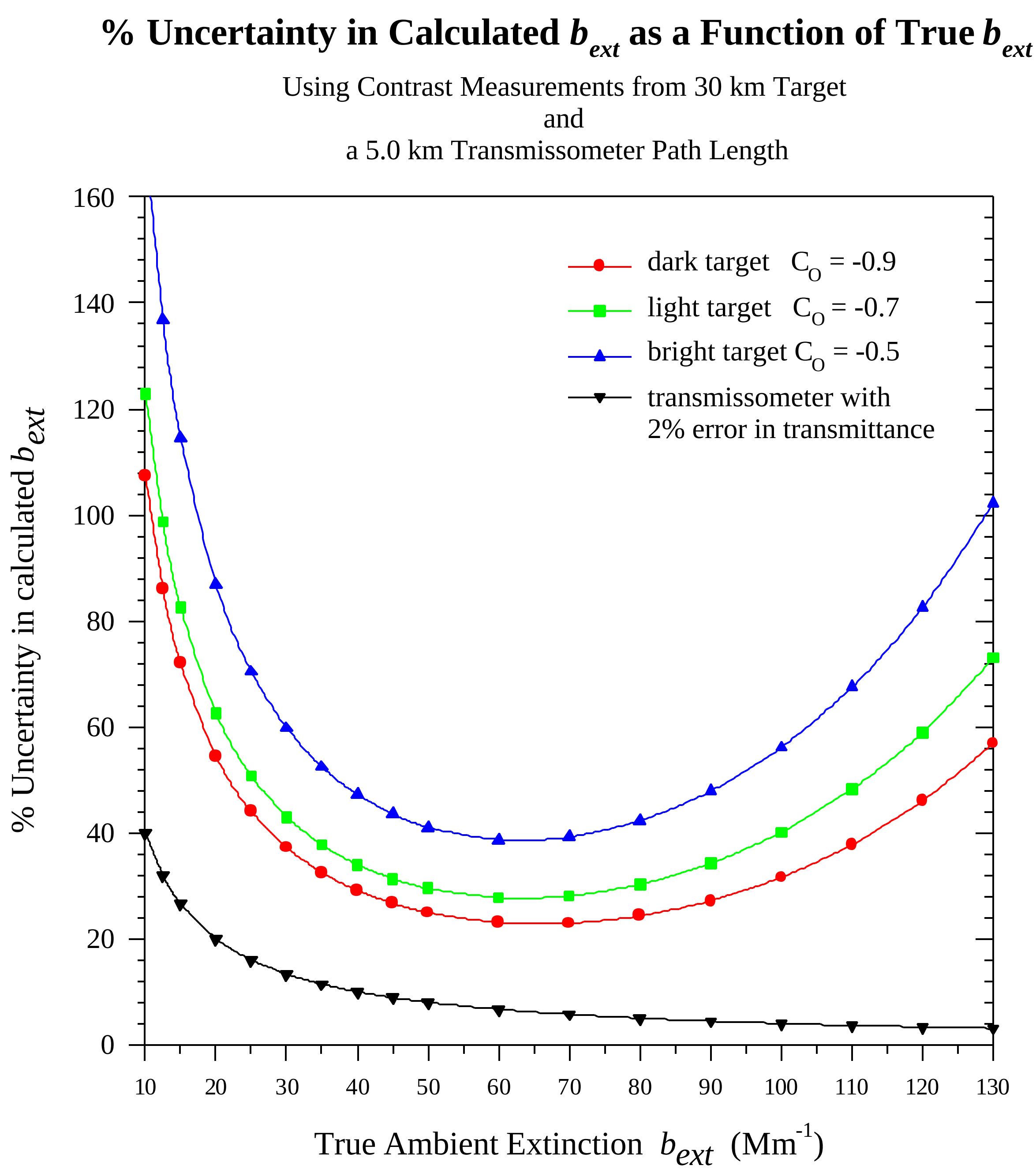
Figure 1



Uncertainty in b_{ext} From Slide Contrast Measurements
bright target C_O = -0.5
light target C_O = -0.7
dark target C_O = -0.9
Uncertainty in Calculated b_{ext} (%)
200
175
150
125
100
75
50
25
0
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
-0.45
-0.50
Apparent Target Contrast: C_r

Figure 2



% Uncertainty in Calculated b_{ext} as a Function of True b_{ext}
Using Contrast Measurements from 30 km Target
and
a 5.0 km Transmissometer Path Length
dark target C_O = -0.9
light target C_O = -0.7
bright target C_O = -0.5
transmissometer with
2% error in transmittance
% Uncertainty in calculated b_{ext}
True Ambient Extinction b_{ext} (Mm^{-1})
0
20
40
60
80
100
120
140
160
10
20
30
40
50
60
70
80
90
100
110
120
130